Ultra-high Energy Predictions of proton-air Cross Sections from Accelerator Data

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We predict $\sigma_{p-\rm air}^{\rm prod}$, the proton–air inelastic production cross section, at pp center-of-mass energies $2 \le \sqrt{s} \le 100000$ TeV, using high energy predictions from a saturated Froissart bound parameterization of accelerator data on forward $\bar{p}p$ and pp scattering amplitudes, together with Glauber theory. The parameterization of the $\bar{p}p$ and pp cross sections incorporates analyticity constraints and unitarity, allowing accurate extrapolations to ultra-high energies. Our predictions are in excellent agreement with cosmic ray extensive air shower measurements, both in magnitude and in energy dependence.

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Introduction. There are now available published p-air inelastic production cross sections[1, 2, 3, 4] ($\sigma_{p-\rm air}^{\rm prod}$) that span the enormous pp cms (center-of-mass system) energy range $2 \le \sqrt{s} \le 100000$ TeV, reaching energies well above the Large Hadron Collider (LHC). Moreover, there are also now available very accurate predictions at cosmic ray energies for the total pp cross section, σ_{pp}, from fits[5] to accelerator data that used adaptive data sifting algorithms[6] and analyticity constraints[7]. However, extracting proton–proton cross sections from published cosmic ray observations of extensive air showers, and vice versa, is far from straightforward[8]. By a variety of experimental techniques, cosmic ray experiments map the atmospheric depth at which extensive air showers develop and measure the distribution of $X_{\rm max}$, the shower maximum, which is sensitive to the inelastic p-air cross section $\sigma_{p-\rm air}^{\rm prod}$. From the measured $X_{\rm max}$ distribution, the experimenters deduce $\sigma_{p-\rm air}^{\rm prod}$. In this note we will compare published values of $\sigma_{p-\rm air}^{\rm prod}$ with predictions made from σ_{pp}, using a Glauber model to obtain $\sigma_{p-\rm air}^{\rm prod}$ from σ_{pp}.

$\sigma_{p-\rm air}^{\rm prod}$ *from the $X_{\rm max}$ distribution: Method I.* The measured shower attenuation length (Λ_m) is not only sensitive to the interaction length of the protons in the atmosphere ($\lambda_{p-\rm air}$), with

$$\Lambda_m = k\lambda_{p-\rm air} = k\frac{14.4 m_p}{\sigma_{p-\rm air}^{\rm prod}} = k\frac{24,100}{\sigma_{p-\rm air}^{\rm prod}}\,, \tag{1}$$

(with Λ_m and $\lambda_{p-\rm air}$ in g cm^{-2}, the proton mass m in g, and the inelastic production cross section $\sigma_{p-\rm air}^{\rm prod}$ in mb), but also depends on the rate at which the energy of the primary proton is dissipated into electromagnetic shower energy observed in the experiment. The latter effect is parameterized in Eq. (1) by the parameter k. The value of k depends critically on the inclusive particle production cross section and its energy dependence in nucleon and meson interactions on the light nuclear target of the atmosphere (see Ref. [8]). We emphasize that the goal of the cosmic ray experiments is $\sigma_{p-\rm air}^{\rm prod}$ (or correspondingly, $\lambda_{p-\rm air}$), whereas in Method I, the *measured* quantity is Λ_m. Thus, a significant drawback of Method I is that one needs a model of proton-air interactions to complete the loop between the measured attenuation length Λ_m and the cross section $\sigma_{p-\rm air}^{\rm prod}$, *i.e.,* one needs the value of k in Eq. (1) to compute $\sigma_{p-\rm air}^{\rm prod}$. Shown in Table I are the widely varying values of k used in the different experiments. Clearly the large range of k-values, from 1.15 for EASTOP[4] to 1.6 for Fly's Eye[1] differ significantly, thus making the *published* values of $\sigma_{p-\rm air}^{\rm prod}$ unreliable. It is interesting to note the monotonic decrease over time in the k's used in the different experiments, from 1.6 used in Fly's Eye in 1984 to the 1.15 value used in EASTOP in 2007, showing the time evolution of Monte Carlo models of energy dissipation in showers. For comparison, Monte Carlo simulations made by Pryke[9] in 2001 of several more modern shower models are also shown in Table I. We see that even among modern shower models, the spread is still significant. The purpose of this letter is a proposal to minimize the impact of model dependence on the determination of $\sigma_{p-\rm air}^{\rm prod}$.

TABLE I: A table of k-values, used in experiments and from Monte Carlo model simulation

Experiment	k
Fly's Eye	1.6
AGASSA	1.5
Yakutsk	1.4
EASTOP	1.15
Monte Carlo Results: C.L. Pryke	
Model	k
CORSIKA-SIBYLL	1.15 ± 0.05
MOCCA–SIBYLL	1.16 ± 0.03
CORSIKA-QGSjet	1.30 ± 0.04
MOCCA–Internal	1.32 ± 0.03

$\sigma_{p-\rm air}^{\rm prod}$ *from the $X_{\rm max}$ distribution: Method II.* The HiRes group[10] has developed a quasi model-free method of measuring $\sigma_{p-\rm air}^{\rm prod}$ directly. They fold into their shower development program a randomly generated exponential distribution of shower first interaction points, and then fit the entire distribution, and not just the trailing edge,

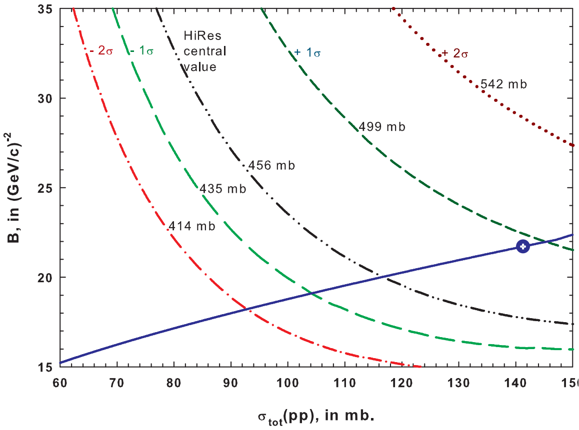

FIG. 1: B dependence on the pp total cross section σ_{pp}. The five curves are lines of constant $\sigma_{p-\rm air}^{\rm prod}$, of 414, 435, 456, 499 and 542 mb—the central value is the published Fly's Eye value, and the others are $\pm 1\sigma$ and $\pm 2\sigma$. The solid curve is a plot of a QCD-inspired fit of B against σ_{pp}, obtained from a $\ln^s$ fit—see text for details. The large dot is the prediction for $\sigma_{p-\rm air}^{\rm prod}$ at $\sqrt{s} = 77$ TeV, the HiRes energy.

as is done in the experiments of Ref. [1, 2, 3, 4]. They obtain $\sigma_{p-\rm air}^{\rm prod} = 460 \pm 14$ (stat) + 39 (syst) − 11 (syst) mb at $\sqrt{s} = 77$ GeV, a result which they claim is effectively model-independent and hence is an absolute determination[10].

Extraction of σ_{pp} from $\sigma_{p-\rm air}^{\rm prod}$. The total pp cross section is extracted from $\sigma_{p-\rm air}^{\rm prod}$ in two distinct steps. First, one calculates the p-air *total* cross section, $\sigma_{p-\rm air}$, from the measured inelastic production cross section using

$$\sigma_{p-\rm air}^{\rm prod} = \sigma_{p-\rm air} - \sigma_{p-\rm air}^{\rm el} - \sigma_{p-\rm air}^{q-\rm el}\,. \qquad (2)$$

Next, the Glauber method[11] is used to transform the measured value of $\sigma_{p-\rm air}^{\rm prod}$ into a proton–proton total cross section σ_{pp}; all the necessary steps are calculable in the theory. In Eq. (2) the measured cross section for particle production is supplemented with $\sigma_{p-\rm air}^{\rm el}$ and $\sigma_{p-\rm air}^{q-\rm el}$, the elastic and quasi-elastic cross section, respectively, as calculated by the Glauber theory, to obtain the total cross section $\sigma_{p-\rm air}$. The subsequent relation between $\sigma_{p-\rm air}^{\rm prod}$ and σ_{pp} critically involves the nuclear slope parameter B, the logarthmic slope of forward elastic pp scattering, $d\sigma_{pp}^{\rm el}/dt$, i.e.,

$$B \equiv \left[\frac{d}{dt}\left(\ln\frac{d\sigma_{pp}^{\rm el}}{dt}\right)\right]_{t=0},. \qquad (3)$$

A plot of B against σ_{pp}, 5 curves of different values of $\sigma_{p-\rm air}^{\rm prod}$, is shown in Fig. 1, taking into account inelastic screening[12]. The reduction procedure from $\sigma_{p-\rm air}^{\rm prod}$ to σ_{pp} is summarized in Ref. [8]. The solid curve in Fig. 1 is a plot of B vs. σ_{pp}, which we will discuss in detail later.

Determination of $\sigma_{pp}(s)$. Block and Halzen[13] have made an analytic amplitude fit that saturates the Froissart bound[14], to both the available high energy total cross section and ρ-value data, where ρ is defined as the ratio of the real to the imaginary portion of the forward scattering amplitude, for both $\bar{p}p$ and pp interactions. For their high energy expressions they used the analytic amplitude form

$$\begin{aligned}\sigma^{\pm}(\nu) &= c_0 + c_1\ln\left(\frac{\nu}{m}\right) + c_2\ln^2\left(\frac{\nu}{m}\right) + \beta_{\mathcal{P}'}\left(\frac{\nu}{m}\right)^{\mu-1}\\ &\quad\pm\,\delta\left(\frac{\nu}{m}\right)^{\alpha-1}, \qquad (4)\\ \rho^{\pm}(\nu) &= \frac{1}{\sigma^{\pm}(\nu)}\left\{\frac{\pi}{2}c_1 + c_2\pi\ln\left(\frac{\nu}{m}\right)\right.\\ &\quad -\beta_{\mathcal{P}'}\cot(\frac{\pi\mu}{2})\left(\frac{\nu}{m}\right)^{\mu-1} + \frac{4\pi}{\nu}f_+(0)\\ &\quad\left.\pm\delta\tan(\frac{\pi\alpha}{2})\left(\frac{\nu}{m}\right)^{\alpha-1}\right\}, \qquad (5)\end{aligned}$$

where the upper sign is for pp and the lower sign is for $\bar{p}p$ scattering, with $\mu = 0.5$, ν is the laboratory energy, $f_+(0)$ is a dispersion relation subtraction constant, and m the proton mass. The 7 real constants $c_0, c_1, c_2, \beta_{\mathcal{P}'}, \delta, \alpha$ and $f_+(0)$ are parameters of the fit. Since at high energies, s, the square of the cms energy, is given by $2m\nu$, we see that their cross section approaches $\ln^2 s$ at high energies, thus saturating the Froissart bound[14].

Using all of the cross sections, σ_{pp} and $\sigma_{\bar{p}p}$, along with all of the ρ-values, $\rho_{\bar{p}p}$ and ρ_{pp}, in the Particle Data Group[15] archive that were in the laboratory energy interval $18.3 \le \nu \le 1.73 \times 10^6$ GeV, i.e., $6 \le \sqrt{s} \le 1800$ GeV, Block and Halzen[13] formed a sieved data set using the sieve algorithm of Ref. [6] to eliminate outliers, which markedly improved their fit[13]. Using 4 analyticity constraints[7], i.e., by fixing both the cross sections $\sigma_{\bar{p}p}$ and σ_{pp} and their laboratory energy derivatives, at $\sqrt{s} = 4$ GeV, they reduced the number of parameters to be fit from 7 to 4 and obtained an excellent fit, which, in turn, constrained pp cross sections at cosmic ray energies to have a relative accuracy $\sim 1-2\%$. Their fits to σ and ρ are shown in Fig. 2(a) and Fig. 2(b), respectively.

Determination of $B(s)$. A QCD-inspired parameterization[16] of forward $\bar{p}p$ and pp scattering amplitudes which is analytic, unitary and fits all data of $\sigma_{\rm tot}$, B and ρ for both $\bar{p}p$ and pp interactions has been made, using 2 analyticity constraints which fix $\sigma_{\bar{p}p}$ and σ_{pp} at $\sqrt{s} = 4$ GeV; see Fig. 2(c) for $B(s)$.

The solid curve in Fig. 1 is a plot of B vs. σ_{pp}, with B taken from the QCD-inspired fit of Ref. [16] and σ_{pp} taken from the Froissart bound fit of Ref. [13]. The large dot corresponds to the value of σ_{pp} and B at $\sqrt{s} = 77$ TeV, the HiRes energy, thus fixing the predicted value of $\sigma_{p-\rm air}^{\rm prod}$ at the HiRes energy.

Obtaining σ_{pp} from $\sigma_{p-\rm air}^{\rm prod}$. In Fig. 3, we have plotted the values of σ_{pp} vs. $\sigma_{p-\rm air}^{\rm prod}$ that are deduced from the

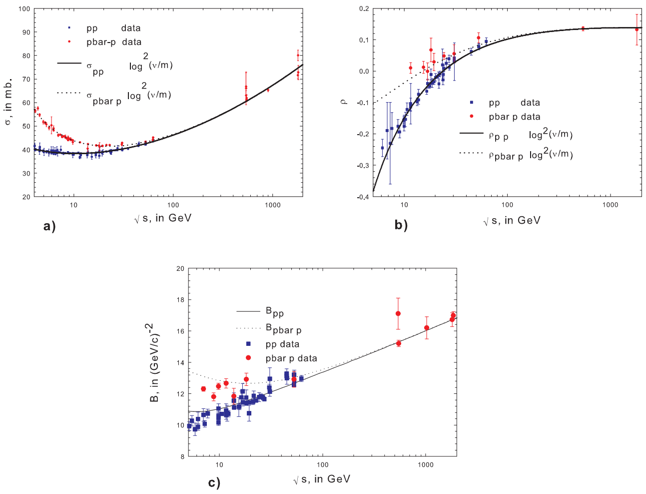

FIG. 2: The saturated Froissart bound fit[13] of total cross section σ_{pp}, ρ *vs.* $\sqrt{s}$, in GeV, for pp (squares) and $\bar{p}p$ (circles) accelerator data: (a) σ_{pp}, in mb, (b) ρ; (c) the nuclear slope B, in GeV^{-2} *vs.* $\sqrt{s}$, in GeV, from a QCD-inspired fit[16].

intersections of the B-σ_{pp} curve with the $\sigma_{p-\mathrm{air}}^{\mathrm{prod}}$ curves in Fig. 1. Figure 3 furnishes cosmic ray experimenters with an easy method to convert their measured $\sigma_{p-\mathrm{air}}^{\mathrm{prod}}$ to σ_{pp}, and vice versa. The percentage error in $\sigma_{p-\mathrm{air}}^{\mathrm{prod}}$ is $\approx 0.4\%$ near $\sigma_{p-\mathrm{air}}^{\mathrm{prod}} = 450$mb, due to the error in σ_{pp} from model parameter uncertainties.

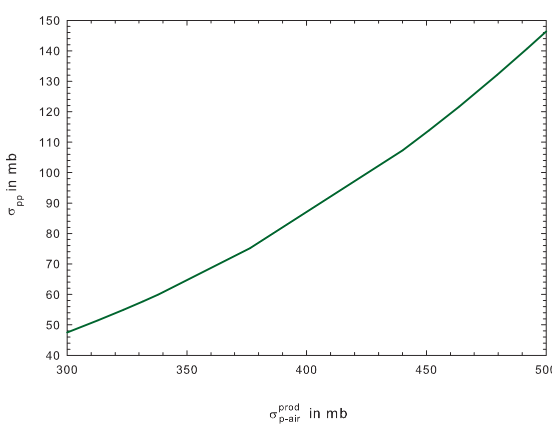

FIG. 3: A plot of the predicted total pp cross section σ_{pp}, in mb *vs.* the measured p-air cross section, $\sigma_{p-\mathrm{air}}^{\mathrm{prod}}$, in mb.

Determining the k value. It is important at this point to recall Eq. (1), $\Lambda_m = k\lambda_{p-\mathrm{air}}$, thus rewinding us of the fact that in Method I, the extraction of $\lambda_{p-\mathrm{air}}$ (or $\sigma_{p-\mathrm{air}}^{\mathrm{prod}}$) from the measurement of Λ_m requires knowing the parameter k. The measured depth $X_{\max}$ at which a shower reaches maximum development in the atmosphere, which is the basis of the cross section measurement in Ref. [1], is a combined measure of the depth of the first interaction, which is determined by the inelastic cross section, and of the subsequent shower development, which has to be corrected for. The model dependent rate of shower development and its fluctuations are the origin of the deviation of k from unity in Eq. (1). As seen in Table I, its values range from 1.6 for a very old model where the inclusive cross section exhibited Feynman scaling, to 1.15 for modern models with large scaling violations.

Adopting the same strategy that earlier had been used by Block et al.[17], we decided to match the data to our prediction of $\sigma_{p-\mathrm{air}}^{\mathrm{prod}}(s)$ in order to extract a *common* value for k. This neglects the possibility of a weak energy dependence of k over the range measured, found to be very small in the simulations of Ref. [9]. By combining the results of Fig. 2 (a) and Fig. 3, we obtain our prediction of $\sigma_{p-\mathrm{air}}^{\mathrm{prod}}$ vs. $\sqrt{s}$, which is shown in Fig. 4. To determine k, we leave it as a free parameter and make a χ^2 fit to *rescaled* $\sigma_{p-\mathrm{air}}^{\mathrm{prod}}(s)$ values of Fly's Eye, [1]AGASSA[2], EAS-TOP[4] and Yakutsk[3], which are the experiments that need a common k-value.

Figure 4 is a plot of $\sigma_{p-\mathrm{air}}^{\mathrm{prod}}$ vs. $\sqrt{s}$, the cms energy in GeV, for the two different types of experimental extraction, using Methods I and II described earlier. Plotted *as published* is the HiRes value at $\sqrt{s} = 77$ TeV, since it is an absolute measurement. We have rescaled in Fig. 4 the published values of $\sigma_{p-\mathrm{air}}^{\mathrm{prod}}$ for Fly's Eye[1], AGASSA[2], Yakutsk[3] and EAS-TOP[4], against our prediction of $\sigma_{p-\mathrm{air}}^{\mathrm{prod}}$, using the *common* value of $k = 1.264 \pm 0.033 \pm 0.013$ obtained from a χ^2 fit, and it is the rescaled values that are plotted in Fig. 4. The error in k of 0.033 is the statistical error of the χ^2 fit, whereas the error of 0.013 is the systematic error due to the error in the prediction of $\sigma_{p-\mathrm{air}}^{\mathrm{prod}}$. Clearly, we have

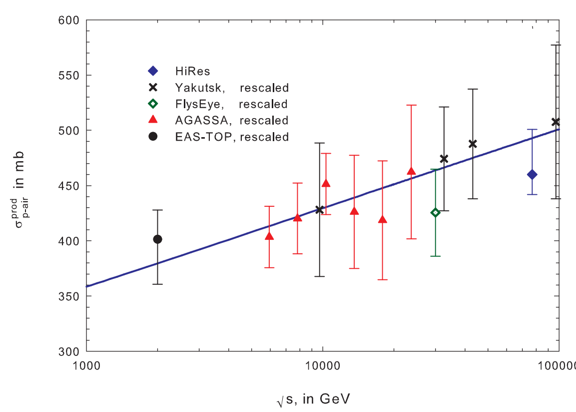

FIG. 4: A χ^2 fit of the *renormalized* AGASA, EASTOP, Fly's Eye and Yakutsk data for $\sigma_{p-\mathrm{air}}^{\mathrm{prod}}$, in mb, as a function of the energy, $\sqrt{s}$, in GeV. The result of the fit for the parameter k in Eq. (1) is $k = 1.263 \pm 0.033$. The HiRes point (solid diamond), at $\sqrt{s} = 77$ GeV, is the model-independent HiRes experiment, which has *not* been renormalized.

an excellent fit, with complete agreement for all experimental points. Our analysis gave $\chi^2 = 3.19$ for 11 degrees of freedom (the low χ^2 is likely due to overestimates of experimental errors). We note that our k-value, $k = 1.264 \pm 0.033 \pm 0.013$, is about halfway between the values of CORSIKA-SIBYLL and CORSIKA-QSGSjet found in the Pryke simulations[9], as seen in Table I.

We next compare our measured k parameter with a direct measurement of k by the HiRes group[18]. They measured the exponential slope of the tail of their X_m distribution, Λ_m and compared it to the p-air interaction length $\lambda_{p-\rm air}$ that they found. Using Eq. (1), they deduced that $k = 1.21 + 0.14 - 0.09$, in agreement with our value, giving us additional experimental confirmation of our method.

Conclusions. Our measured k value, $k = 1.264 \pm 0.033 \pm 0.013$, agrees very well with the k-value measured by the HiRes group, at the several parts per mil level, and in turn, they both agree with Monte Carlo model simulations at the 5–10 part per mil level.

It should be noted that the EASTOP[4] cms energy, 2 TeV, is essentially identical to the top energy of the Tevatron collider, where there is an *experimental* determination of $\sigma_{\bar{p}p}$[19], and consequently, no necessity for an *extrapolation* of collider cross sections. Since their value of $\sigma_{p-\rm air}^{\rm prod}$ is in excellent agreement with the predicted value of $\sigma_{p-\rm air}^{\rm prod}$, this serves to anchor our fit at its low energy end. Correspondingly, at the high end of the cosmic ray spectrum, the absolute value of the HiRes experimental value of $\sigma_{p-\rm air}^{\rm prod}$ at 77 TeV—which requires *no knowledge* of the k parameter—is also in good agreement with our prediction, anchoring the fit at the high end. Thus, our $\sigma_{p-\rm air}^{\rm prod}$ predictions, which span the enormous energy range, $2 \le \sqrt{s} \le 100000$ TeV, are completely consistent with *all* of the cosmic ray data, for both magnitude and energy dependence.

In the future, we look forward to the possibility of confirming our analysis with the high statistics of the Pierre Auger Collaboration[20], as well as confirming the prediction of 107.3 ± 1.2 mb for the total pp cross section[13] at the LHC energy of 14 TeV.

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